SELF-OSCILLATING PROCESS OF HEAT EXCHANGE WITH PERIODIC INTENSITY

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The limit of the stability of a heat carrier flow against small "density wave"-type perturbations is found. It is shown that stabilization of flow sets in on increase in the volume heat capacity and also in the amplitude of the heat-transfer coefficient fluctuations.

When a temperature-dependent heat carrier flows in a heated channel, the manifestation of a specific low-frequency instability with respect to density waves is possible [1]. This form of instability, associated with the effects of interaction and delay on lengthwise propagation of the perturbations of flow rate, density, and pressure, is characteristic for two-phase media [2, 3] and for a single-phase fluid in a supercritical region of pressures [4].

An approximate analytical method of the investigation of stability with respect to density waves has been developed in [5, 6]. A flow of helium in a supercritical region of pressures was considered for the limiting case of infinitely small values of the volume heat capacity of a wall. In the present work a generalization of the analysis of [5, 6] is made taking account of the effect of thermal conjugation with a wall on the limit of stability.

Similarly to [5], a case is considered in which pressure losses in a lengthwise direction can be neglected in comparison with those on the inlet and outlet restrictors. Then the heat carrier flow in a channel is described by one-dimensional continuity and energy equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0\,, \tag{1}$$

$$\rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial z} = q_v\,. \tag{2}$$

It is assumed that the specific volume of the heat carrier depends linearly on the enthalpy:

$$\frac{dv}{dh} = a = \text{const}\,. \tag{3}$$

For a homogeneous two-phase flow equality (3) is trivial and is satisfied exactly [1]; for helium in a supercritical region of pressures it is satisfied approximately [5, 6].

The use of (3) allows one to eliminate the density and enthalpy from Eqs. (1) and (2) and reduce them to the form

$$\frac{\partial u}{\partial z} = a q_v\,, \tag{4}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial z} = a q_v v\,. \tag{5}$$

It is assumed that the presence of small perturbations of the velocity (flow rate) of a heat carrier at the inlet to the channel is the sole perturbing factor [2, 3]. There are thus

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a) the condition of the invariability of the sum of pressure drops on the inlet and outlet restrictors:

$$\Delta p_1 + \Delta p_2 = k_1 \rho_1 u_1^2 + k_2 \rho_2 u_2^2 = \text{const}\,; \tag{6}$$

b) the condition of the constancy of the heat carrier enthalpy at the inlet to the channel (downstream from the inlet restrictor):

$$h_1 = \text{const}\,.$$

Taking account of Eq. (3), the enthalpy boundary condition is replaced by the specific-volume boundary condition:

$$v_1 = \text{const}\,. \tag{7}$$

The rate, specific volume, and the density of heat sources are represented as the superposition of stationary and fluctuating quantities:

$$u = \bar{u} + u',\quad v = \bar{v} + v',\quad q_v = \bar{q}_v + q_v'.$$

The main flow $\bar{u}$, $\bar{v}$ also must satisfy relations (4) and (5). Subtracting the averaged equations from the total ones, we obtain a system of equations for fluctuations of the quantities u', v', and q_v':

$$\frac{\partial u'}{\partial z} = a q_v', \tag{8}$$

$$\frac{\partial v'}{\partial t} + \bar{u}\frac{\partial v'}{\partial z} + u'\frac{d\bar{v}}{dz} = a\bar{q}_v v' + a q_v' \bar{v}\,. \tag{9}$$

The boundary conditions for Eqs. (8) and (9) follow from boundary conditions (6) and (7):

$$v_1' = 0\,, \tag{10}$$

$$\Delta p_1' + \Delta p_2' = 0\,. \tag{11}$$

The solutions of the equations for the fluctuations of the quantities (8), (9) are sought in the form

$$\frac{u'}{\bar{u}} = A_u(z)\exp(\Omega\tilde{t}) = A_u(z)\exp(\gamma\tilde{t})\,[\cos(\beta\tilde{t}) + i\sin(\beta\tilde{t})]\,,$$

$$\frac{v'}{\bar{v}} = A_v(z)\exp(\Omega\tilde{t}) = A_v(z)\exp(\gamma\tilde{t})\,[\cos(\beta\tilde{t}) + i\sin(\beta\tilde{t})]\,.$$

In accordance with the method developed in [7, 8] for investigation of the processes of heat exchange with periodic intensity, for the considered one-dimensional nonstationary case the fluctuations of the density of heat sources and velocity can be connected approximately by the linear relation

$$\frac{q_v'}{\bar{q}} = \chi\frac{u'}{\bar{u}}\,, \tag{12}$$

where χ is the "coefficient of conjugation," whose specific form is given below. We will introduce the scale of frequency:

$$\omega_0 = a\bar{q}_v$$

and the dimensionless longitudinal coordinate:

$$y = 1 + \omega_0 z/\bar{u}_1 .$$

Here, the quantity a, defined by relation (3), characterizes the "degree of expansion" of the heat carrier along the length of the channel:

$$a = \frac{v_2 - v_1}{h_2 - h_1} .$$

The solutions of Eqs. (4) and (5) for the main flow $\bar{u}$, $\bar{v}$ are

$$\frac{\bar{v}}{\bar{v}_1} = \frac{\bar{u}}{\bar{u}_1} = y .$$

The complex amplitudes of the fluctuations of velocity and specific volume are described by the equations

$$\frac{dA_u}{dy} = \chi A_u , \tag{13}$$

$$\frac{dA_v}{dy} + \frac{\Omega - 1}{y} A_v + \frac{A_u}{y} = 0 , \tag{14}$$

where $\Omega = \omega/\omega_0$ is the dimensionless frequency. The boundary conditions for Eqs. (13), (14) follow from Eqs. (10), (11):

$$A_{v1} = 0 , \tag{15}$$

$$2EKA_{u1} + 2A_{u2} - A_{v2} = 0 . \tag{16}$$

Here $E = \rho_1/\rho_2 = v_2/v_1$, $K = \Delta\bar{p}_1/\Delta\bar{p}_2 = k_1/(Ek_2)$.

The solutions of Eqs. (13) and (14), which satisfy boundary conditions (15), will have the form

$$A_u = Cy^{\chi} , \tag{17}$$

$$A_v = \frac{C(1 - \chi)}{\Omega + \chi - 1} (y^{1-\Omega} - y^{\chi}) , \tag{18}$$

Determining from Eqs. (17) and (18) the values of the amplitudes A_u, A_v at the inlet and outlet of the channel and substituting them into the boundary condition (16), we eliminate the constant C and obtain the following dispersion relation

$$2 (EKA_{v1} + E^{\chi}) (\Omega + \chi - 1) = (1 - \chi) (E^{1-\Omega} - E^{\chi}) . \tag{19}$$

Letting $\Omega = y + i\beta$ in Eq. (19) and dividing the real and imaginary parts of the resulting equality, we obtain a system of two transcendental equations that connect the quantities K, E, γ, and β:

$$\begin{aligned} 2 (\gamma + \chi - 1) (EK + E^{\chi}) &= (1 - \chi) [E^{1-\gamma} \cos (\beta \ln E) - E^{\chi}] , \\ 2\beta (EK + E^{\chi}) &= - (1 - \chi) E^{1-\gamma} \sin (\beta \ln E) . \end{aligned} \tag{20}$$

At the given values of the parameters K, E the imaginary part (β) of the complex frequency Ω corresponds to the frequency of periodic fluctuations; the real part (γ) of the frequency Ω determines the form of flow, i.e., stable or unstable.

In [5, 6], when analyzing a flow of helium in a supercritical region of pressures, it was assumed that the volume heat capacity of the wall was vanishingly small. Physically this means that fluctuations of heat sources are equal to zero ($q'_v = 0$).

We shall change from the general case (20) to the limiting case [5, 6]; for this purpose we equated to zero the coefficient of conjugation in equality (12):

$$2(\gamma - 1)(EK + 1) = E^{1-\gamma}\cos(\beta \ln E) - 1\,,$$

$$2\beta(EK + 1) = -E^{1-\gamma}\sin(\beta \ln E)\,. \tag{21}$$

In the other limiting case ($\chi = 1$) Eq. (20) yields

$$\gamma E(K + 1) = 0\,,\ \ \beta E(K + 1) = 0\,. \tag{22}$$

The system of equations (22) has a trivial solution:

$$\gamma = \beta = 0\,,$$

which corresponds to the absolute stability of flow.

On the other hand, the system of equations (21), investigated in [5, 6], determines a certain limit of stability in the coordinates $E(K)$. Thus, the limiting evaluations show that with increase in the coefficient of conjugation χ the stability of the flow must increase.

An approximate analytical solution of dispersion relation (20) has the following form:

$$\beta = \frac{(3/2)\,\pi}{\ln E}\,, \tag{23}$$

$$\frac{F \ln F}{3\pi\Phi} - 1 = KF\,. \tag{24}$$

Here $F = E^{1-\chi}$, $\Phi = E^{\gamma}$ are the generalized parameters.

Equality (23) means that the frequency of periodic fluctuations of perturbations depends only on the parameter of the expansion of flow between the inlet and outlet restrictors ($E = v_2/v_1 \ge 1$). Equality (24) describes three possible situations: a) stability ($\gamma < 0$; $\Phi < 1$); b) instability ($\gamma > 0$; $\Phi > 1$); c) stability limit ($\gamma = 0$; $\Phi = 1$).

From Eq. (24) it follows that in the absence of throttling at the exit ($k_2 = \Delta \bar{p}_2 = 0$; $K \to \infty$) the stability of flow increases under other equal conditions. Of greatest interest for specific applications is the limit of stability, whose equation follows from Eq. (24) at $\Phi = 1$:

$$\frac{F \ln F}{3\pi} - 1 = KF\,. \tag{25}$$

Assigning in Eq. (25) the generalized parameter $F = E^{1-z}$, it is possible to calculate the parameter $K = \Delta\bar{p}_1/\Delta\bar{p}_2$ at the limit of stability. In the absence of throttling at the exit from the channel ($k_2 = \Delta\bar{p}_2 = 0$; $K \to \infty$), relation (25) has the asymptotics:

$$F \approx \exp(3\pi K) \to \infty\,. \tag{26}$$

From relation (26) it follows that the flow will preserve stability for any (infinitely large in the limit) value of the parameter of flow expansion E.

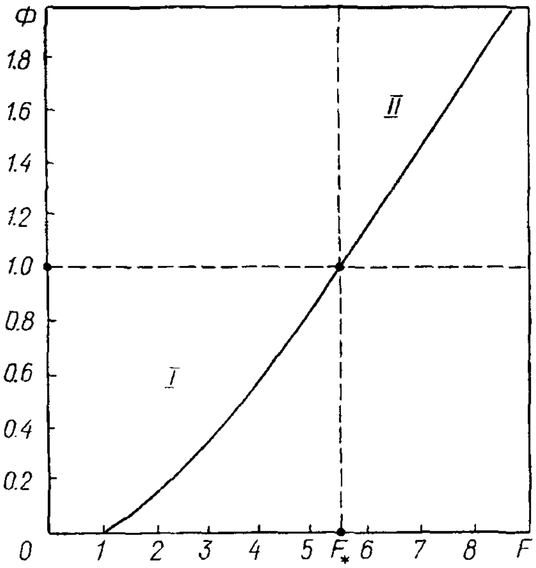

Fig. 1. Solution of the dispersion relation of a linear problem of stability for $K = 0$: *I*) region of stability; *II*) region of instability.

We will consider now solution (24) for the other limiting case, when there is no throttling at the inlet to the channel ($k_1 = \Delta \bar{p}_1 = K = 0$).

Assuming in Eq. (24) $K = 0$, we will obtain

$$\Phi = \frac{F \ln F}{3\pi}. \tag{27}$$

Equality (27) describes three possible situations (Fig. 1): a) stability ($1 \le F < F_*$; $0 \le \Phi < 1$); b) instability ($F_* < F < \infty$; $1 < \Phi < \infty$); c) limit of stability ($F = F_*$; $\varphi = 1$). Here $F_* \approx 5.52$ is the value of the parameter F at the limit of stability.

Thus, analysis of the variants, which are limiting with respect to the parameter K, shows that with increase in throttling at the inlet (or with decrease in throttling at the outlet) the stability of flow increases and vice versa. Below we will consider the practically most important case of$K = 0$, which is characterized by the smallest stability. The sought dependence of the parameter of flow expansion at the limit of stability for $K = 0$ on the coefficient of conjugation can be conveniently written using Eq. (25) in the following form:

$$E = F_*^n, \quad n = \frac{1}{1 - \chi}. \tag{28}$$

For a further investigation of relation (28) it is necessary to obtain an expression in an explicit form for the coefficient of conjugation χ in relation (12). For this purpose we shall use the method developed in [7, 8] for investigating processes of heat exchange with periodic intensity, according to which the effect of heat carrier flow on the wall is considered to be equivalent to the assignment to the heat exchange surface of a heat transfer coefficient, which periodically changes with time relative to its mean value:

$$\alpha(\tilde{t}) = \bar{\alpha}(1 + b \cos \beta \tilde{t}). \tag{29}$$

Then, from the solution of the heat conduction equation with a periodic boundary condition of the 3rd kind we can find the needed parameters of the conjugated conduction-convection problem, in particular, the value of n. For low-frequency thermohydraulic vibrations in a long heated, by internal heat sources, tube, which were considered in [4-6], the method of [7,8] yields

$$n = 1 + \frac{2B}{f(1 + f)}. \tag{30}$$

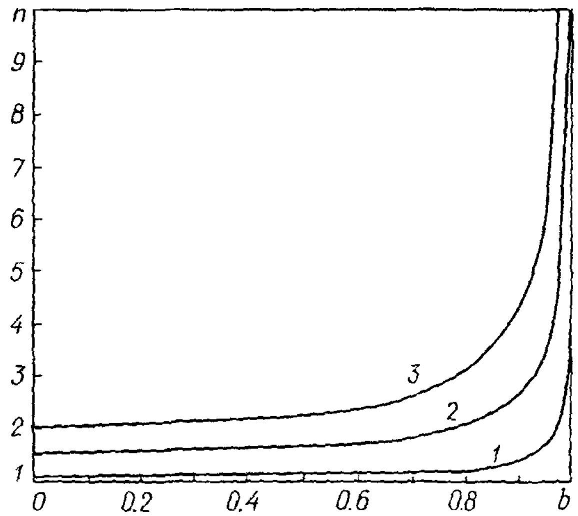

Fig. 2. Extrapolation of the solution of the dispersion relation to a nonlinear region: 1) $B = 0.1$, 2) 0.7, 3) 1.0.

Here $f = \sqrt{1 - b^2}$; $B = (\frac{\rho_w c_w \delta_w \omega_0}{\bar{\alpha}})^2$.

Thus, the limit of stability of the flow is determined from the amplitude of the fluctuations of the heat transfer coefficient b, frequency of fluctuations ω_0, mean value of the heat transfer coefficient $\bar{\alpha}$, and the volume heat capacity of the wall $(\rho_w c_w \delta_w)$.

For the considered "linear approximation" ($b << 1$, $f = 1$) relation (30) yields

$$n = 1 + B. \tag{31}$$

Assuming in Eq. (31) that $B = 0$, we arrive at the limiting case, considered in [5, 6], of zero volume heat capacity of the wall:

$$n = 1\,; \quad E_* = F_* = 5.52\,.$$

With increase in the parameter B the stability of the flow increases:

$$E_* \to \infty \quad \text{when} \quad B \to \infty\,.$$

We will assume now that solution (23), (24) obtained in linear approximation can be used also at finite values of amplitude b ("nonlinear approximation"). Then, relation (30) describes increase in the flow stability with increase in b:

$$E_* \to \infty \quad \text{when} \quad b \to 1 \quad (n \to \infty)\,. \tag{32}$$

Relations (30), (32) make it possible to suggest the following scenario of the development of self-oscillations for unstable processes of heat exchange with periodic intensity:

a) Linear instability. Small perturbations of parameters introduced into a flow increase exponentially. The rate of increase is determined by the quantity γ and can be calculated from relation (27):

$$\gamma = \frac{\ln \Phi}{\ln E} > 0\,.$$

b) Nonlinear instability. Fluctuating quantities become commensurable with the means:

$$u' \approx \bar{u}\,; \quad v' \approx \bar{v}\,; \quad q_v' \approx \bar{q}_v'\,; \quad b \approx 1\,.$$

Since, in this case, according to Eq. (32), the flow at $b = 1$ is absolutely stable, relation (30) determines such a value of $b_* < 1$ at which the flow parameters correspond to the limit of stability ($\gamma = 0$; $\Phi = 1$).

c) Stable nonlinear self-oscillations When $b > b_*$, the flow has the "stability margin." The amplitudes of the fluctuations of parameters in accordance with Eq. (27) decay exponentially:

$$\gamma = \frac{\ln \Phi}{\ln E} < 0 ,$$

so that at $b = b_*$ the flow again arrives at the limit of stability. When $b < b_*$, the flow has a "deficit of stability." Then, an increase in perturbations again brings the flow to the limit of stability, $b = b_*$. The computational dependences of the parameter $n = (1 - \chi)^{-1}$ on the amplitude of the fluctuations of the heat transfer coefficient b are presented in Fig. 2. At a fixed value of b the curves with a larger value of the parameter B correspond to larger values of the flow expansion parameter E. At a fixed value of the parameter B and with increase in the amplitude b the parameter n increases, and this again means an increase in the flow stability. As seen from Fig. 2, the functions $n(b)$ have a distinct "boundary-layer" character: a sharp increase in the stability occurs in the region with $b \to 1$.

The proposed model of a self-oscillatory process of heat exchange with periodic intensity can be used to calculate the limit of stability with respect to perturbations of the type of "density wave" with account taken of thermal conjugation with a wall.

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NOTATION

t, time; z, longitudinal coordinate; ρ, heat carrier density; h, specific enthalpy of the heat carrier; u, velocity of the heat carrier; v, specific volume of the heat carrier; α, heat transfer coefficient; b, amplitude of the fluctuations of the heat transfer coeffcient; q_v, density of heat sources; Δp, pressure losses on a restrictor; k, coefficient of hydraulic drag of a restrictor; A_u, amplitude of velocity fluctuations; A_v, amplitude of specific volume fluctuations; $\Omega = \gamma + i\beta$, complex frequency of fluctuations; χ, coefficient of conjugation; $\omega_0 = a\bar{q}_v$, scale of frequency; $\bar{t} = \omega_0 t$, dimensionless time; y, dimensionless longitudinal coordinate; E, parameter of the expansion of the heat carrier between the inlet and outlet constrictors; K, parameter of pressure drops on the inlet and outlet restrictors; ρ_w, density of the material of the wall; c_w, specific heat of the wall; δ_w, thickness of the wall. The bar above denotes the value averaged for the period of fluctuations; the stroke denotes a pulsating value. Subscripts: 1, inlet to the channel; 2, outlet from the channel; w, wall; *, limit of stability.

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